Solow's golden rule application for brazilian economy

Marcelo Miranda de Melo¹ Guaracyane Campelo² Cleycianne Almeida³

Resumo: Esse trabalho acadêmico utilizou a teoria da regra de ouro do modelo de Solow na economia brasileira. Uma função de produção modificada foi introduzida no modelo com os seguintes argumentos: capital e crédito. Dados econômicos nacionais foram comparados envolvendo Brasil, parceiros selecionados na América Latina e também o grupo dos BRICs. Variáveis macroeconômicas como: Produto Interno Bruto (PIB) (preços constantes) % variação, Taxa Nacional de Poupança Bruta (% PIB), Inflação % variação foram comparadas entre os países selecionados. O Brasil tem a menor taxa de poupança (% PIB) entre países latinos americanos. Também entre países latinos americanos o Brasil apresentou uma das menores taxas de crescimento econômico perdendo apenas para o México. Entre os BRICs a China apresentou com folga o maior crescimento econômico com a maior taxa de poupança (% PIB) e a mais baixa taxa de inflação. Brasil e África do Sul parecem ter dados similares. O modelo de Solow foi desenvolvido e utilizando técnicas de otimização encontrou-se a taxa de poupança ótima para a economia brasileira. Utilizou-se análise de

¹ Civil Engineer(UFC), MSc in Construction Management(UMIST-U.K.), PhD in Economics(UFC), Finance and Economics Professor at UFC.

² Economist (UFC), MSc in Economics (UFC), PhD in Economics (UFC), Finance and Economics Professor at UFC.

³ Economist (UFC), MSc in Economics (UFPE), PhD in Economics (UFPE), Finance and Economics Professor at UFC.

regressão com dados abrangendo o período de 1991 a 2012 usando PIB, Crédito e Capital. A taxa de poupança ótima foi encontrada em s = 42,94(%PIB). Atualmente a taxa de poupança brasileira está bem abaixo da taxa ótima, s = 13,0 (%PIB), o que em grande parte explica o baixo crescimento brasileiro. O Brasil deve elevar sua taxa de poupança no sentido de obter um crescimento econômico sadio e sustentável.

Palavras-Chave: Crescimento econômico. Taxa de poupança. Regra de ouro.

Classificação JEL: O11. O41.

Abstract: This academic work used Solow's Golden Rule theory into the Brazilian economy. A modified production function was introduced applying capital and credit as arguments. Comparative country economic figures were presented involving Brazil and selected Latin American economic partners and also BRIC team. Macroeconomic variables such as: Gross Domestic Product – GDP (constant prices) % change, Gross National Savings – GNS (%GDP), Inflation % change were compared across selected countries. Brazil has the lowest GNS (%GDP) in comparison to Latin American countries. Also in comparison to Latin American team, Brazil also presented one of the lowest economic growths losing only for Mexico. Among the BRICs China presented by far the highest economic growth with also the highest GNS (%GDP) and the lowest inflation rate. Brazil and South Africa seems to have similar figures. The Solow's model was developed and using optimizing techniques the optimum savings rate for Brazilian economy was found. Applying regression analysis covering the time span of 1991 up to 2012 with GDP, Credit and Capital was found the optimum saving rate (%GDP), s = 42,94. Present Brazil's saving rate is very far behind the optimum, s = 13%, what in great extend explains the low Brazilian economic growth. Brazil should increase its saving rate in order to have a healthy and sustainable economic growth.

Keywords: Economic Growth. Savings Rate. Golden Rule.

JEL Classification: O11. O41.

I Introduction

The Brazilian economy has faced outstanding challenges since the American Crisis. Perhaps the major outcome was low economic growth. The international market, since 2008, was not the appropriate trading ground, especially for developing countries and also major countries due to economic recession in important players in Europe and in the US. Brazil was forced to look inside and improve its domestic market in order to overcome the American Crisis.

The Brazilian government applied Keynesian Theory adopting some economic measures such as: reduced interest rates, increased credit availability, reduced bank compulsory deposits and also applied fiscal policy measures in order to improve economic sectors activity in the short run. Besides those measures, the Brazilian government has foster government expenditures. However the major increase was in the administration sphere and not preferably into the infrastructure expenditures. The size of the State into the economy has increased provoking unnecessary intervention in the private sector.

Probably the most relevant economic measure adopted was the increase in credit availability into the Brazilian economy. Brazil presented a sharply increase in credit from 23,8% of GDP (December 2008) up to 55,8% of GDP (February 2014). Consume of families has increased dramatically since then. However, few measures were adopted to improve production capacity what induced to an increase in inflation in the medium term.

The need to improve credit availability was necessary due to low savings rate of the Brazilian economy. There is no dough that improving credit availability was imperative at that time; however its positive impact was in a short term period. The most upsetting feature of the Brazilian economy seems to be low economic growth rate.

Most important economic growth models point out the appropriate savings rate as a relevant factor for healthy and sustainable economic growth. In Brazil the savings rate is still low in comparison to selected countries, especially in the developing team.

Due to remarkable increase in credit availability and major focus on domestic market, especially in the consume account; the Brazilian production function seems to has changed. This fact suggests that the recommended production function for the Brazilian economy should include the credit argument. This research applied the Solow's Golden Rule using a modified production function in order to find out the savings rate for the Brazilian economy that maximizes consume. What savings rate that maximizes consume in Brazil? What its impacts in economic growth?

The production function adopted for the Brazilian economy was Cobb-Douglas type with capital and credit as arguments, both in *per capita* figures. Both capital and credit presents concave functions and Inada conditions. This proposed production function also presents decreasing returns of scale.

2 Research background

Schenk–Hoppé (2002), analyzed the dependence of average consumption on the saving rate in a one-sector neoclassical Solow growth model. They showed that the long-run behavior of the stochastic capital intensity, and hence average consumption along any sample path, is uniquely determined by a random fixed point that depends continuously on the saving rate. This result enable they to prove the existence of a golden-rule saving rate that maximizes average consumption *per capita*.

Mankiw, Phelps & Romer (1995), developed a model that takes **s**, n and d as exogenous. This approach parsimoniously turns the Solow model into a rigorous general-equilibrium model. In particular, the economy can reach a steady state with the capital stock greater than what Edmund Phelps called the Golden Rule.

Zhigang & Zhangyong (2003), developed a thesis that discusses the optimum consumption and the decision mechanism of saving with reference to Solow's economic growth model and Ramsey-Cass-Koopmans model, deriving the golden rule of judging the dynamic efficiency of macro-economic operation and modifying the standards of golden rule.

Edwards (1996), presents a theoretical and empirical assessment of the determinants of savings rates, with special emphasis on Latin American savings rates. The study is based on international comparisons, using data from 36 countries for 1970–1992. He concludes that low Latin American savings are due to the magnitudes of their determinants, rather than structural differences.

Blomstrom, Lipsey & Zejan (1996), examined shares of fixed capital formation in GDP and rates of economic growth for more than 100 countries over successive 5-year periods between 1965 and 1985 to determine the direction of causality between them. They conclude that high rates of fixed capital formation accompany rapid growth in *per capita* income, but they found no evidence that fixed investment is the only or main source of ignition for economic growth.

Jappelli & Pagano (1994) conclude that in the context of an overlapping-generations model, they showed that liquidity constraints on households: (i) raise the saving rate,(ii) strengthen the effect of growth on saving.

Bosworth, Collins & Reinhart (1999) compared China and Brazil as portfolio capital receivers. China, the largest developing-country recipient of FDI in the 1990s, obtained very little portfolio capital or lending, while Brazil was the largest receiver. They conclude that such inflows can raise growth rates by supplementing domestic saving, thereby raising the rate of capital accumulation.

Levine & Zervos (1998), pointed out growth indicators-measures of the rate of economic growth: capital accumulation, productivity growth, and private saving. They highlighted private saving as a key factor for economic growth.

Bresser-Pereira & Gala (2007), developed a formalization of the critique of the growth with foreign savings strategy. They explain that although medium income countries are capital poor, current account deficits (foreign savings), financed either by loans or by foreign direct investments, will not usually increase the rate of capital accumulation or will have little impact on it in so far as current account deficits will be associated with appreciated exchange rates, artificially increased real wages and salaries and high consumption levels. They indicate a sustainable policy of increasing private savings.

Fry (1980), presented a quantitative estimate of the cost of financial repression in developing countries. Here, financial repression is interpreted as the technique of holding institutional interest rates (particularly deposit rates of interest) below their market equilibrium levels. He concludes that credit availability is an important determinant not only of new investment but also of capacity utilization of the entire capital stock.

Masson, Bayoumi & Samiei (1998), showed a broad set of possible determinants of private saving behavior. They used data for a large sample of industrial and developing countries. Both time-series and cross sectional estimates are obtained. Results suggest that there is a partial offset on private saving of changes in public saving and (for developing countries) in foreign saving, that demographics and growth are important determinants of private saving rates, and that interest rates and terms of trade have positive, but less robust, effects.

Horioka & Wan (2006), in this paper, the authors conducted a dynamic panel analysis of the determinants of the household saving rate

in China using a life cycle model and panel data on Chinese provinces for the 1995–2004 period from China's household survey. They found that China's household saving rate has been high and rising and that the main determinants of variations over time and over space therein are the lagged saving rate, the income growth rate, (in many cases) the real interest rate, and (in some cases) the inflation rate.

Loayza & Schmidt-Hebbel (2000), concluded that saving rates display considerable variation across countries and over time. In this paper, authors investigate empirically the policy and nonpolicy factors behind these saving disparities using a large, cross-country, time-series data set and following an encompassing approach including a number of relevant private saving determinants. They also concluded that the income growth rate, the real interest rate and the lagged saving rate are relevant factors to explain saving rates across countries.

3 Brazilian macroeconomic variables

In order to present an overview of the Brazilian Macroeconomic dynamics, selected variables data were collected in IMF and IPEADATA such as:

- a. Gross Domestic Product-GDP (constant prices) % of change;
- b. Gross National Savings-GNS % of change;
- c. Inflation (average consumer prices) % of change;
- d. Credit % of GDP.

The GDP, GNS and Inflation data coved the time span from 1980 to 2014 and it is presented in the graphs below. The Credit data shows annual increase from 2008 up to 2013.

From graph 1 it is clear that the Brazilian GDP presented a very volatile behavior in the last 35 years. Especially in the last 3 years there is a tendency of decreasing figures. Gross Savings shows also a decreasing behavior and in the last 3 years figures below 15% GDP. Very high inflation was presented in the Brazilian economy for a long time. Since 1994 from the establishment of Plano Real, inflation became to decrease sharply and continue in one digit figures. In order to face the American Crisis the Brazilian government conducted a greater participation of credit operations in GDP as a major countercyclical measure. From graph 4 it's evident that credit participation almost double from 2008 to late 2013 as % of GDP. In recent times this never happen to the Brazilian economy.



Graph 1: Gross domestic product % changes

Source: IMF-World Economic Outlook Database, October 2014.



Graph 2: Gross national savings % GDP

Source: IMF-World Economic Outlook Database, October 2014.



Graph 3: Inflation (average consumer prices) % Changes

Source: IMF-World Economic Outlook Database, October 2014.



Graph 4: Credit in % of Brazilian GDP

Source: Central Bank of Brazil (www.bcb.org.br).

4 Brazilian economy in comparison to economic partners

This research also made relevant comparisons between Brazil and Latin American and BRIC economic partners.

In Latin American selected countries were pointed out and data collected from IMF. Besides Brazil, data from the following countries were collected: Argentina, Chile, Colombia and Mexico. The macroeconomic variables collected from the last 20 years were: Gross Domestic Product - GDP (%), Gross National Savings - GNS (% GDP) and Inflation. Table 1 presents macroeconomic variables in average of the last 20 years of the Brazilian Latin American partners.

Variable	Brazil	Argentina	Chile	Colombia	Mexico
GDP (%)	3,36	3,76	5,05	3,82	2,74
GNS (%GDP)	17,23	18,36	23,5	19,01	23,68
Inflation (%)	9,44	7,63	4,93	10,42	10,34

Table 1: Macroeconomic Variables of Latin American Partners (average %p.a.)

Source: IMF-World Economic Outlook Database, October 2014.

Brazil has the lowest GNS (%GDP) in comparison to Latin American countries. Chile presented the highest economic growth in average from the last 20 years with the lowest inflation rate and with high GNS (%GDP) in comparison to Latin American team. Brazil also presented one of the lowest economic growths losing only for Mexico. Colombia showed the highest inflation rate with relevant economic growth and with good GNS (%GDP). Brazil appears to be behind its Latin American economic partners especially in relation to economic growth even thought inflation in under control.

Among the developing countries Brazil should be compared to BRIC members such as: Russia, India, China and South Africa. In table 2 macroeconomic variables of BRIC team are presented.

Variable	Brazil	Russia	India	China	South Africa
GDP (%)	3,36	2,08	7,06	10,53	3,22
GNS (%GDP)	17,23	29,36	29,83	47,10	16,28
Inflation (%)	9,44	86,12	7,79	4,93	6,86

Table 2: Macroeconomic Variables of BRIC members (average %p.a.)

Source: IMF-World Economic Outlook Database, October 2014.

China presented by far the highest economic growth with also the highest GNS (% GDP) and the lowest inflation rate. India also presented a remarkable economic growth with suitable GNS (% GDP). Russia showed the lowest economic growth with by far the highest inflation rate. Brazil and South Africa seems to have similar figures.

From section 2 Brazil appears to have control over inflation but presents an unsustainable economic growth rate. Another relevant macroeconomic variable is the GNS (% GDP) which in the Brazilian case is continuing decreasing for below 15% level. Credit availability is continuing increasing and approaching 60% Credit/GDP level. Credit policy has being successful at the beginning of the rising policy in order to face harmful symptoms from the American Crisis. The credit policy seems to be a short term successful measure, not provoking increasing tendency for long term economic growth. One of the major reasons for the low economic growth in Brazil seems to be the GNS (% GDP). With appropriate GNS (% GDP) Brazil tends to increase its GDP in a sustainable fashion.

5 The economy and the production function

This economy is accomplished by consume and investment as formulated in equation 1. Savings (S) will be automatically linked to investment (I).

Y = C + I	(1)
$I \equiv S$	(2)

The production function initially is formed as F (K, L, CR) where K represents capital, L labor and CR credit. Dividing all arguments by labor we get in equation 3: F (K/L, L/L, CR/L) or F (K/L, 1, CR/L). Using *per capita* figures the equation 3 becomes F (k, Cr). Finally the production function can be formulated as in equation 4.

$$F(k,Cr) \qquad (3)$$

$$y = k\alpha Cr\beta \ 0 < \alpha < 1, \ 0 < \beta < 1 \qquad (4)$$

Credit availability depends on income as stated as in equation 5.

$$Cr = \gamma y \quad 0 < \gamma < 1$$
 (5)

The production function arguments also present concave function which can be found using first and second derivatives as follows:

$$F_k > 0, F_{kk} < 0, F_{cr} > 0, F_{crcr} < 0.$$

Due to lack of appropriate infrastructure, high taxes, and other structural characteristics; the Brazilian economy has presented low productivity rate. This way in this economy, it's assumed that $\alpha + \beta < 1$, performing a decreasing returns of scale economy.

Substituting equation 5 into 4 we get equation 6.

$$y = k\alpha (\gamma y)\beta \tag{6}$$

Developing equation 6 and changing variable using $\gamma\beta = \theta$ we get equation (7)

$$y = (\Theta k^{\alpha})^{(1/1-\beta)} \tag{7}$$

According to Solow's Model in the steady state condition k = 0, there is no increase or decrease of capital accumulation. The accumulation of capital is expressed in equation 8. S represents the savings rate, n the rate of increase of labor and d is the depreciation rate.

 $k = sf(k) - (n+d)k \qquad (8)$

The golden rule for capital accumulation is the way to find out the optimum amount of capital for social welfare in an economy. The long term social welfare maximization is reached when the level of *per capita* consume is maximum. Therefore, the golden rule for capital accumulation consists in the savings rate finding. The savings rate determines the investment rate and subsequently the consume rate.

The optimum amount of capital (k^*) leads to a steady state in which *per capita* consume is maximum. This situation can be expressed in equation 9 as follows:

$$C^* = f(k^*) - sf(k^*)$$
 (9)

Applying steady state condition k=0 in equation 8 using equation 7 we get k^* .

$$k^* = [(se^{1/1-\beta})/(n+d)]^{1-\beta/1-(\alpha+\beta)} (10)$$

The consume equation is derived from equations 1 and 7 as follows:

$$C = \Theta^{1/1-\beta} k^{\alpha/1-\beta} - (n+d)k \quad (11)$$

Applying k* in equation 11 and maximizing consume with respect to s (savings rate) we find the level of s that maximizes *per capita* consume. After calculations we get:

$$s = \alpha/1 - \beta \tag{12}$$

6 Empirical results

Regression analysis was applied using equation 4 in order to estimate α and β parameters. Using natural log figures we can reach equation 13.

$$lny = \alpha lnk + \beta lnCr$$
 (13)

Research data was collected from IPEADATA covering the time span of 1991 up to 2012 using *per capita* figures of GDP(y), Credit(Cr) and Capital(k). Regression analysis result is presented in table 3.

Lnpib	Coef.	Std. Err.	t	P> t	95% conf	interval
Lncred	0,0140575	0,0027467	5,12	0,0000	0,0083086	0,0198063
Lnfbk	0,4234411	0,0889268	4,76	0,0000	0,2373151	0,6095672
const	5,471	0,5027646	10,88	0,0000	4,418702	6,523299

Table 3: Regression analysis model result

Source: Own research.

According to table 3, α and β parameters were found and statistically significant. The already mentioned parameters can be expressed by: $\alpha = 0.4234411$ and $\beta = 0.0140575$. Both parameter values passed in the t test with very robust results. R squared and adjusted R squared values showed the following results: 75,14% and 72,53%; a remarkable predictive potentiality.

Using equation 12 and including parameters values already found, the level for savings % GDP that maximizes social welfare is reached, according to Solow's Golden Rule. Calculating s = 0,429478 or 42,9478% GDP.

From the Golden Rule steady state, simulations were performed using Mathlab and changing the level of credit into the Brazilian economy. The level of savings rate or GNS (%GDP) remained stable in 42,9478% GDP. The results obtained from the steady state are as follows:

 $\gamma = 0.557 \ \alpha = 0.4234411 \ \beta = 0.0140575 \ n = 0.0086 \ d = 0.10$

Increasing credit level (γ) by 5,74% greater than steady state the following variables: product, consume and capital will also increase in close measures. All results are presented in table 4.

Product	0,14%
Consume	0,1389%
Capital	0,1404%

Table 4: Simulation analysis (increasing credit)

Source: Own research.

Decreasing credit level (γ) by -10,23% smaller than steady state the following variables: product, consume and capital will also decrease in close measures. All results are presented in table 5.

Table 5: Simulation analysis (decreasing credit)

Product	-0,27%
Consume	-0,2716
Capital	-0,2688

Source: Own research.

These results confirm Solow's Golden Rule achievements and highlight the relevance of an appropriate savings rate and credit level for the entire economy.

7 Final remarks

This research applied Solow's Golden Rule in the Brazilian economy using as a production function a Cobb-Douglas type including as parameters capital and credit. Economic data has showed a very volatile and also a decreasing behavior in Brazil's GDP figures. A low savings rate should be one remarkable reason for this behavior. Nowadays (March-2015) Brazil's savings rate is below 15% GDP and falling. Credit availability is also a relevant feature of the Brazilian economy as well. In the last decade, credit availability considering % GDP figures almost doubled in Brazil. The value of γ is close to 60% indicating greater credit participation into the Brazilian economy since 2008. Moreover, the result $\alpha + \beta < 1$, also confirmed decreasing returns of scale for the Brazilian economy.

Developing the Solow's economic growth model the optimum savings rate was found. According to Solow's Golden Rule this rate maximizes social welfare by maximizing consume. This figure was approximately 43% GDP and very high in comparison to Brazil's present savings rate of around 13% GDP. Brazilian economy should increase, in an urgent fashion, its savings rate in order to improve economic growth.

One relevant factor for private savings is the real interest rate in Brazil. Due to long periods of high inflation the Brazilian Central Bank is applying high real interest rates, however taxation is still very high what diminishes financial investment attractiveness from private sector of the economy. Brazilian population is getting older and this feature may influence the savings rate behavior. This particular characteristic was also mentioned by Masson, Bayoumi & Samiei (1998).

Reviewing macroeconomic country data in tables 1 and 2, it's also clear that countries with high savings rate %GDP tend to have higher economic growth rates. Paying attention to China 's figures with GNS = 47,10%GDP and with average economic growth rate of 10,52%p.a.; it's evident the relevance of savings rate. Even in the Latin American circle, Chile is a very good example with GNS = 23,5%GDP and with average economic growth rate above 5%p.a..This research also confirms conclusions of Schenk–Hoppé (2002), Zhigang & Zhangyong (2003), Levine & Zervos (1998) and Loayza, Schmidt-Hebbel & Servén (2000).

Brazil should adopt measures to encourage the saving mechanism such as: reducing taxation from financial investments, apply positive real interest rates, offer a saving education program to especially low age population, etc. According to this academic research, there is an outstanding gap from the present savings rate to its optimum. Also making comparisons with other countries the lack of appropriate savings rate seems to be a major factor for low economic growth in Brazil.

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